Exam 1 Math 6952 Fall 2026

Name:__________________________

Directions: Show all of your work and justify all of your answers.

(10) 1. Suppose that $f : [a,b] \to \mathbb{R}$. Define what it means for f to be Riemann integrable on $[a,b]$.

(10) 2. State the Cauchy Criterion.

(10) 3. State the Squeeze Theorem for Integrals.

(10) 4. State the Fundamental Theorem of Calculus

5. Answer each of the following as true or false. You need not justify your answers.

(10) a. If $f : [a,b] \to \mathbb{R}$ is continuous, then $f \in \mathcal{R}[a,b]$.

True.

(10) b. If $f : [a,b] \to \mathbb{R}$ is bounded, then $f \in \mathcal{R}[a,b]$.

False.

(10) c. If $f \in \mathcal{R}[a,b]$, then f is bounded.

True.

(20) 6. Define $f : [0,1] \to [0,1]$ by

$$f(x) = \begin{cases} 1 & \text{if } x = \frac{1}{2} \\ 0 & \text{if } x \neq \frac{1}{2} \end{cases}$$

Use the **definition of the Riemann integral** to show that $\int_0^1 f = 0$.

Proof: Let $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta < \varepsilon$ and suppose that $\|\dot{\mathcal{P}}\| < \delta$. Since at most two of the tags equal $\frac{1}{2}$ and $f(x) = 0$ for $x \neq \frac{1}{2}$,

$$|S(f,\dot{\mathcal{P}}) - 0| = |S(f,\dot{\mathcal{P}})| \leq 2\delta < 2\varepsilon$$

as desired. ■

(10) 7. Define $f_0 : [0,1] \to [0,1]$ by $f_0(x) = 0$ for all $x \in [0,1]$. For each $n \in \mathbb{N}$, let $f_n : [0,1] \to [0,1]$ such that $\int_0^1 f_n = \frac{1}{n}$ and suppose that $g : [0,1] \to [0,1]$ such that $g(x) \leq f_n(x)$ for all $n \in \mathbb{N}$ and for all $x \in [0,1]$. Prove that $g \in \mathcal{R}[0,1]$ and find $\int_0^1 g$.

***Proof*:** We employ the Squeeze Theorem. Let $\varepsilon > 0$ be given. Choose $N \in \mathbb{N}$ such that $\frac{1}{N} < \varepsilon$. Since $f_0(x) \leq g(x) \leq f_N(x)$ for all $x \in [0,1]$ and $|\int_0^1 f_0 - \int_0^1 f_N| = |\int_0^1 f_N| = \frac{1}{N} < \varepsilon$, $g \in \mathcal{R}[0,1]$ by the Squeeze Theorem. Also, since $0 \leq \int_0^1 g \leq \int_0^1 f_n = \frac{1}{n}$ for all $n \in \mathbb{N}$, $\int_0^1 g = 0$. ■

(20) 8. Define $f : [0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} n & \text{if } x = \frac{1}{n} \text{ for some } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

Sketch the graph of f. Explain why $f \notin \mathcal{R}[0,1]$.

The function is not Riemann integrable on [0,1] because it is unbounded.

(20) 9. Suppose that $f : [a,b] \to \mathbb{R}$ has a continuous derivative, $f(a) = 1$ and $f(b) = -3$. Compute $\int_a^b f'$.

By the Fundamental Theoreom of Calculus,

$$\int_a^b f' = f(b) - f(a) = -3 - 1 = -4$$