Theorem 1: For all \( x \in \mathbb{R} \), \( \sin^2 x + \cos^2 x = 1 \).

Corollary 2: For all \( x \in \text{dom}(\tan) \cap \text{dom}(\sec) \), \( \tan^2 x + 1 = \sec^2 x \).

Corollary 3: For all \( x \in \text{dom}(\cot) \cap \text{dom}(\csc) \), \( \cot^2 x + 1 = \csc^2 x \).

Theorem 4: For all \( x, y \in \mathbb{R} \),

(i) \( \sin(x + y) = \sin x \cos y + \cos x \sin y \);

(ii) \( \cos(x + y) = \cos x \cos y - \sin x \sin y \).

Corollary 5 (Double-Angle Formulas): For all \( x \in \mathbb{R} \),

(i) \( \sin(2x) = 2\sin x \cos x \);

(ii) \( \cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \).

Corollary 6 (Half-Angle Formulas): For all \( x \in \mathbb{R} \),

(i) \( \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} \);

(ii) \( \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} \).

Theorem 7: For all \( x, y \in \mathbb{R} \),

(i) \( \sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)] \);

(ii) \( \cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)] \).

(iii) \( \sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)] \).

Theorem 8: Let \( a, b \in \mathbb{R} \) with \( a \leq b \) and suppose that \( f : [a, b] \to \mathbb{R} \) such that \( f' \) is continuous. The length of the curve \( y = f(x) \) from \( a \) to \( b \) is \( \int_a^b \sqrt{1 + [f'(x)]^2} \, dx \).

Theorem 9: Suppose that \( f \) is a function such that \( f' \) is continuous and \( f(x) \geq 0 \) for all \( x \in [a, b] \). Also, let \( \mathcal{R} \) be the region bounded by the curves \( y = f(x) \), \( x = a \), \( x = b \), and the \( x \)-axis. Then the surface area of the solid formed by revolving \( \mathcal{R} \) about the \( x \)-axis is

\[
\int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} \, dx.
\]

Theorem 10: Suppose that \( f : \mathbb{R} \to \mathbb{R} \) is continuous and non-negative on the closed interval \([a, b]\) and let \( \mathcal{R} \) be the region bounded by the curves \( x = a \), \( x = b \), \( y = 0 \), and \( y = f(x) \). Also, let \( A \) be the area of \( \mathcal{R} \). Then the center of mass of \( \mathcal{R} \) is \((\overline{x}, \overline{y})\) where

\[
\overline{x} = \frac{1}{A} \int_a^b x f(x) \, dx \quad \text{and} \quad \overline{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2 \, dx.
\]

Theorem 11: Suppose that \( f : \mathbb{R} \to \mathbb{R} \) and \( g : \mathbb{R} \to \mathbb{R} \) are continuous and \( f(x) \leq g(x) \) on the closed interval \([a, b]\) and let \( \mathcal{R} \) be the region bounded by the curves \( x = a \), \( x = b \), \( y = f(x) \), and \( y = g(x) \). Also, let \( A \) be the area of \( \mathcal{R} \). Then the center of mass of \( \mathcal{R} \) is \((\overline{x}, \overline{y})\) where

\[
\overline{x} = \frac{1}{A} \int_a^b x [g(x) - f(x)] \, dx \quad \text{and} \quad \overline{y} = \frac{1}{A} \int_a^b \frac{1}{2} [(g(x))^2 - (f(x))^2] \, dx.
\]
**Theorem 12 (Divergence Test):** If the series $\sum_{n=1}^{\infty} x_n$ converges, then $\lim_{n \to \infty} x_n = 0$.

**Theorem 13 (Comparison Test):** Suppose that $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ are series such that $0 \leq x_n \leq y_n$ for all $n \in \mathbb{N}$. If $\sum_{n=1}^{\infty} y_n$ converges, then $\sum_{n=1}^{\infty} x_n$ converges. If $\sum_{n=1}^{\infty} x_n$ diverges, then $\sum_{n=1}^{\infty} y_n$ diverges.

**Theorem 14:** Let $a, r \in \mathbb{R}$ with $a, r \neq 0$.

(i) If $|r| < 1$, then $\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}$; 

(ii) if $|r| \geq 1$, then $\sum_{n=0}^{\infty} ar^n$ diverges.

**Theorem 15 (Limit Comparison Test):** Suppose that $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ are series such that $x_n \geq 0$ and $y_n \geq 0$ for all $n \in \mathbb{N}$. If $\lim_{n \to \infty} \frac{x_n}{y_n} = L \in \mathbb{R}$ such that $L > 0$, then $\sum_{n=1}^{\infty} x_n$ converges if and only if $\sum_{n=1}^{\infty} y_n$ converges.

**Theorem 16 (Integral Test):** Suppose that $f : [1, \infty) \to \mathbb{R}$ is a continuous decreasing function with $f(x) > 0$ for all $x \in [1, \infty)$. Then the series $\sum_{n=1}^{\infty} f(n)$ converges if and only if the improper integral $\int_{1}^{\infty} f(x) \, dx$ converges.

**Theorem 17 (Alternating Series Test):** Let $\{x_n\}_{n=1}^{\infty}$ be a decreasing sequence of positive numbers. Then $\sum_{n=1}^{\infty} (-1)^n x_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} x_n$ converge if and only if $\lim_{n \to \infty} x_n = 0$.

**Theorem 18 (The Ratio Test):** Let $\{x_n\}_{n=1}^{\infty}$ be a sequence. If $\lim_{n \to \infty} \left| \frac{x_{n+1}}{x_n} \right| = L < 1$, then the series $\sum_{n=1}^{\infty} x_n$ converges absolutely. If $\lim_{n \to \infty} \left| \frac{x_{n+1}}{x_n} \right| = L > 1$ or $\lim_{n \to \infty} \left| \frac{x_{n+1}}{x_n} \right| = \infty$, then the series $\sum_{n=1}^{\infty} x_n$ diverges.
Theorem 19 (The Root Test): Let \( \{x_n\}_{n=1}^{\infty} \) be a sequence. If \( \lim_{n \to \infty} \sqrt[n]{|x_n|} = L < 1 \), then the series \( \sum_{n=1}^{\infty} x_n \) converges absolutely. If \( \lim_{n \to \infty} \sqrt[n]{|x_n|} = L > 1 \) or \( \lim_{n \to \infty} \sqrt[n]{|x_n|} = \infty \), then the series \( \sum_{n=1}^{\infty} x_n \) diverges.

Theorem 20: Let \( \sum_{n=0}^{\infty} c_n (x - a)^n \) be a power series in \( x - a \) with coefficient set \( \{c_n\}_{n=1}^{\infty} \). Then one of the following is true.

(i) The power series \( \sum_{n=0}^{\infty} c_n (x - a)^n \) converges only if \( x = a \).

(ii) The power series \( \sum_{n=0}^{\infty} c_n (x - a)^n \) converges for all \( x \in \mathbb{R} \).

(iii) There is a number \( R > 0 \) such that the power series \( \sum_{n=0}^{\infty} c_n (x - a)^n \) converges absolutely for all \( x \in (a - R, a + R) \) and diverges for all \( x \notin [a - R, a + R] \).

Theorem 21: Let \( a \in \mathbb{R} \) and \( \{c_n\}_{n=0}^{\infty} \) be a sequence of real numbers such that the power series \( \sum_{n=0}^{\infty} c_n (x - a)^n \) has radius of convergence \( R > 0 \) or \( \infty \) and interval of convergence \( I \). Define \( f : I \to \mathbb{R} \) by \( f(x) = \sum_{n=0}^{\infty} c_n (x - a)^n \). Then

(i) \( f \) is differentiable on \( \text{int}(I) \) and \( f'(x) = \sum_{n=1}^{\infty} c_n (x - a)^{n-1} \) which has the same radius of convergence as \( \sum_{n=0}^{\infty} c_n (x - a)^n \);

(ii) \( \int f(x) \, dx = \sum_{n=0}^{\infty} c_n \frac{(x - a)^{n+1}}{n+1} + C \) which has the same radius of convergence as \( \sum_{n=0}^{\infty} c_n (x - a)^n \).
**Theorem 22:** Suppose that \( \mathcal{C} \) is a smooth curve with parametric equations \( x = f(t) \) and \( y = g(t) \). Then the slope of the tangent line to \( \mathcal{C} \) at a point \((x_0, y_0) = (f(t_0), g(t_0))\) is \( \frac{g'(t_0)}{f'(t_0)} \) provided \( f'(t_0) \neq 0 \).

**Theorem 23:** Let \( \mathcal{C} \) be a plane curve with parametric equations \( x = f(t) \) and \( y = g(t) \). Also, suppose that \( f : [a, b] \to \mathbb{R} \) and \( g : [a, b] \to \mathbb{R} \) are differentiable and for all \( t_1, t_2 \in [a, b] \) with \( t_1 < t_2 \), \( (f(t_1), g(t_1)) = (f(t_2), g(t_2)) \) if and only if \( t_1 = a \) and \( t_2 = b \). Then the area of the region enclosed by \( \mathcal{C} \) is 
\[
\int_a^b f(t)g'(t)\,dt = \int_a^b (g(t)f'(t))\,dt.
\]

**Theorem 24:** Let \( \mathcal{C} \) be a smooth plane curve with parametric equations \( x = f(t) \) and \( y = g(t) \). Also, suppose that \( f : [a, b] \to \mathbb{R} \) and \( g : [a, b] \to \mathbb{R} \) such that for all \( t_1, t_2 \in [a, b] \) with \( t_1 < t_2 \), \( (f(t_1), g(t_1)) = (f(t_2), g(t_2)) \) implies that \( t_1 = a \) and \( t_2 = b \). Then the arc length of \( \mathcal{C} \) is 
\[
\int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt.
\]

**Theorem 25:** Let \( \mathcal{C} \) be a smooth plane curve with parametric equations \( x = f(t) \) and \( y = g(t) \). Also, suppose that \( f : [a, b] \to \mathbb{R} \) and \( g : [a, b] \to [0, \infty) \) such that for all \( t_1, t_2 \in [a, b] \) with \( t_1 < t_2 \), \( (f(t_1), g(t_1)) = (f(t_2), g(t_2)) \) implies that \( t_1 = a \) and \( t_2 = b \). Then the surface area of the solid formed by rotating \( \mathcal{C} \) about the \( x \)-axis is 
\[
\int_a^b 2\pi g(t)\sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt.
\]

**Theorem 26:** Let \( a, b \in \mathbb{R} \) with \( 0 \leq a < b \leq 2\pi \). Also, suppose that \( f : [a, b] \to \mathbb{R} \) is continuous and \( f(\theta) \geq 0 \) for all \( \theta \in [a, b] \). Then the area of the region bounded by the polar curves, \( \theta = a \), \( \theta = b \), and \( r = f(\theta) \) is 
\[
\frac{1}{2} \int_a^b [f(\theta)]^2\,d\theta = \frac{1}{2} \int_a^b r^2\,d\theta.
\]

**Theorem 27:** Let \( f : [a, b] \to \mathbb{R} \) such that \( f' \) is continuous and suppose that \( \mathcal{C} \) is the graph of the curve with polar equation \( r = f(\theta) \). Then the length of \( \mathcal{C} \) is 
\[
\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta = \int_a^b \sqrt{r^2 + \left(\frac{df}{d\theta}\right)^2}\,d\theta.
\]

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<td>32 ft/s(^2)</td>
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<td>m(^2)</td>
<td>ft(^2)</td>
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<td>( F )</td>
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